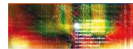

What students can learn about statistics
using paperfrogs



Markus Vogel, University of Education Heidelberg



Maputo - Mosambik, 10.10.2011

Classroom snapshot

In the morning somewhere in a classroom ...

Which paper frog will jump better – the big or the small one?



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Classroom snapshot

Excerpt of a students' discussion

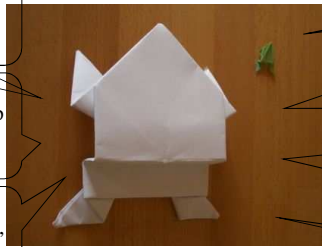
„The heaviness of the big one seems to be advantageous because flying more stable“

„By the way, what means „better“? Is it the furthest jump or the average of several jumps?“

„But there are also really bad jumps of the little one possible, beside that the big one has more powerful legs.“

„I think so, because you only have to press more powerful“

...“



„the small frog is more lightweight.“

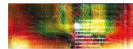
„More stable does not necessary mean that this frog is jumping better.“

„Even on average the little one is better because of the really far jumps.“

„But the big one is weighting more, thus the more powerful legs will not assure success.“

„By the way, how will be the measurement done?“

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Classroom snapshot

schedule and discussion points for classroom activities:

hypothesizing about variables:

bigness, weight, technique of paper folding, dexterity of the frog-let-jumping person

procedure and measurement :

one jump is not enough, but how many are needed? How to measure?
By whom is to be measured – always by the same person or changing?

descriptive analyses:

mean or median? how to describe variation – numerical or/and graphical? Using which representations resp. graphics

inferential analyses:

what can be predicted? what can be generalized beyond the data?

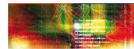
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Overview

1. Snapshot of classroom activities
2. Classroom activities 5-7 grade
3. Classroom activities 8-10 grade
4. Classroom activities 11-12 grade
5. Why should we do this – curricula aspects
6. Ideas behind
7. conclusion



Classroom activities 5-7 grade

Reducing complexity:

Concentrating on one variable only – the students proposed to compare paper weight

Defining research question:

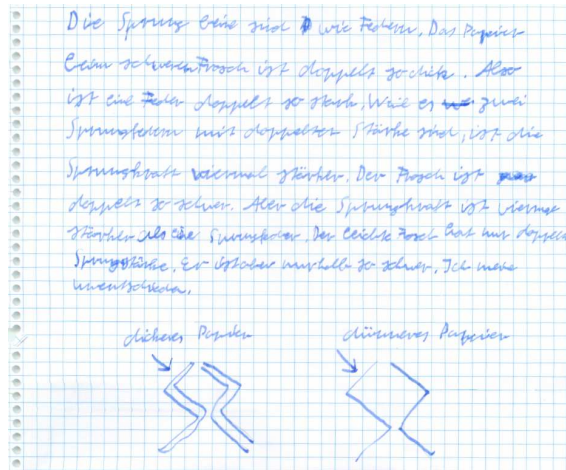


Which frog will jump further?

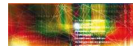


Classroom activities 5-7 grade

Hypothesizing:



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Classroom activities 5-7 grade

Procedure and measurement:

- How to measure - like it is usual done at the shotput or at the long jump?
- How many measurements per paperfrog?
- Who measures, who let the frog jumping, who is logging the data?



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Classroom activities 5-7 grade

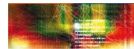
Data:

lightweight paperfrog:

48; 18,5; 32; 51; 51; 17; 54; 52; 41; 18;
18; 42; 13; 29; 42; 51; 43; 37; 46; 27;
10; 34; 11; 15; 52; 47; 57; 54; 10; 39

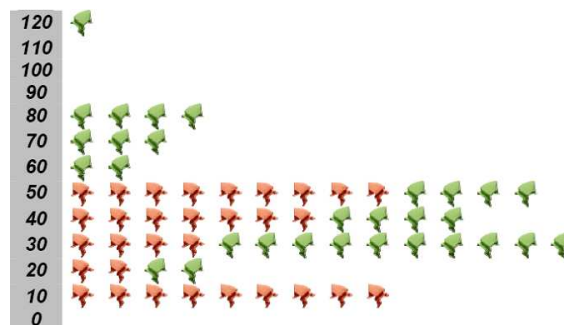
heavyweight paperfrog:

41; 66; 59; 27; 36; 51; 27; 34; 59; 33;
64; 72; 89; 42; 39; 56; 85; 30; 46; 36;
129; 89; 32; 71; 42; 72; 32; 35; 34; 89



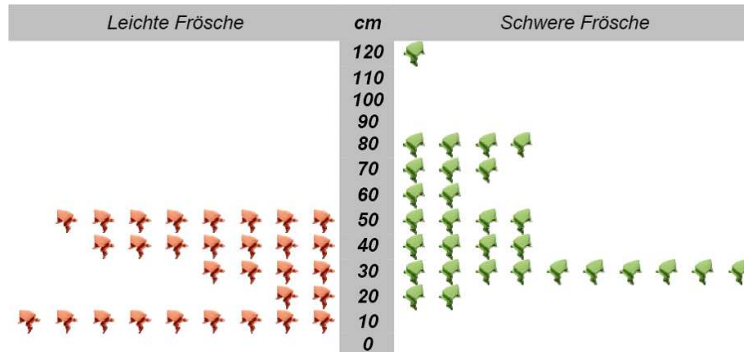
Classroom activities 5-7 grade

Descriptive statistics: How to represent graphically? This way ...

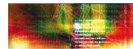


Classroom activities 5-7 grade

Descriptive statistics: or this way ...

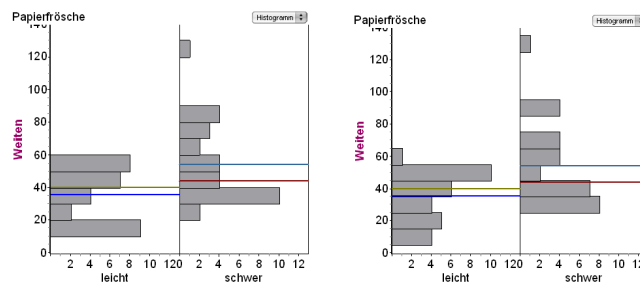


Students are asked to verbalize what they see in the graphic with regard to center and distribution of the data



Classroom activities 5-7 grade

Descriptive statistics: same data, but different histograms



Heavyweight frogs: mean = 51,3cm, median = 44cm
 Lightweight frogs: mean = 35,3cm, median = 40cm

Deviation was estimated and described qualitatively only

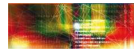


Classroom activities 5-7 grade

Aspects of “inferential statistics”:

Within the 5-7 graders we had only estimations for predictions resp. generalization.

But the students had carefully to argue on why it is difficult to generalize on base of this data-set.



Überblick

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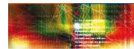


Classroom activities 8-10 grade

Phases of defining research question, hypothezing, procedure and measurement, data collecting and graphical analysis were similar carried through within the 8-10 grade classes.

The special focus here was on the question: How can the outcome of the experimental data-set be judged, how can generalisations be deduced?

The goal was to implement the idea of a permutation test.



Classroom activities 8-10 grade

Inferential statistics (informal, computerbased)

The question is: Within the real experiment there was an mean difference about 16cm. When we repeat this experiment, how often a mean difference of 16cm (or even more) will be reached?

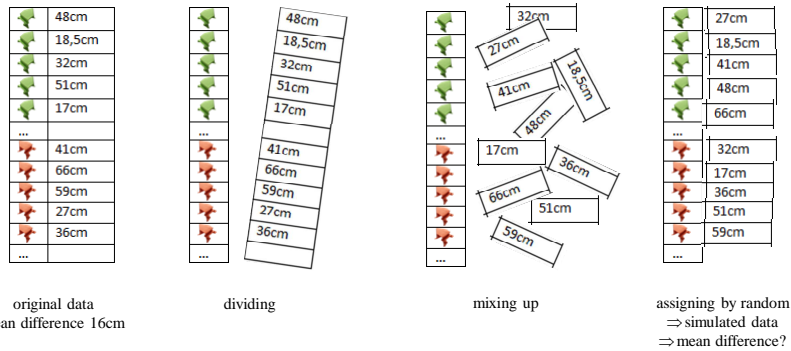
Problem: It is practically seen not possible to repeat such an experiment a thousand times. The idea of a permutation test is instead of that to use simulations based on the original data-set



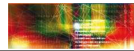
Classroom activities 8-10 grade

Inferential statistics (informal, computerbased)

Before using the computer students have to simulate by hand, so they can see and understand what the computer will repeat very fast.



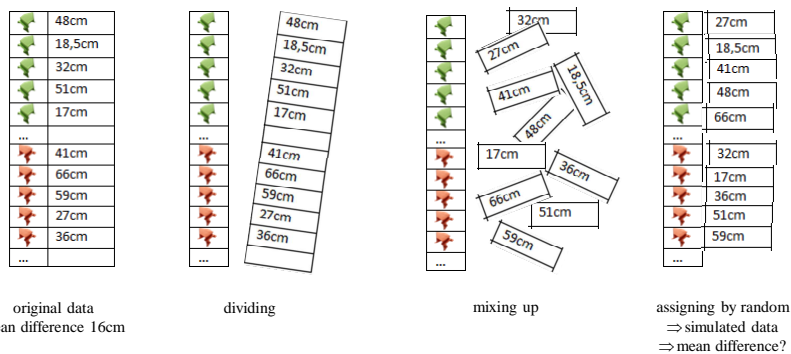
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Classroom activities 8-10 grade

Inferential statistics (informal, computerbased)



After done by hand several times using the computer



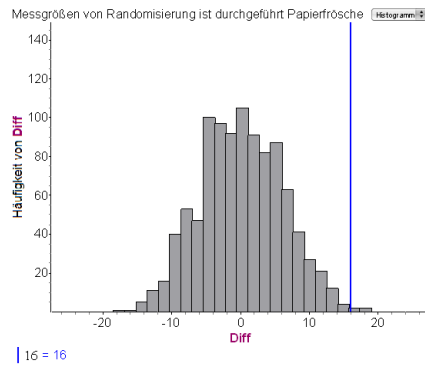
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Classroom activities 8-10 grade

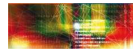
Inferential statistics (informal, computerbased)



Result and interpretation:

Caused by random within 1000 times only 3 times the difference was higher or equal to 16cm.

Thus, the probability of reaching 16cm of the original data-set caused by random seems to be very low and thus, there must be a trend.



Überblick

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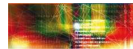


Classroom activities 11-12 grade

After having reduced complexity by considering one influencing variable only the focus is enlarged on the two variables paper weight and paper size.

The special focus here was on the question: How can the outcome of an experimental bivariate data-set be judged, how can generalisations be deduced?

The long-term objective was to prepare the idea of describing interdependencies between two variables (here of course, they are not) and judge them.



Classroom activities 11-12 grade

Data:

	kleine Frösche	große Frösche
leichte Frösche	72, 63, 70, 100, 62, 90, 84, 40, 67, 74, 83, 83, 102, 53, 95, 83, 72, 79, 75, 42, 88, 93, 65, 64, 83, 80, 78, 78, 88, 96 $\bar{x} = 76,7; s = 15,3$	31, 63, 37, 56, 50, 58, 45, 66, 77, 69, 54, 33, 65, 58, 55, 47, 37, 40, 34, 25, 19, 27, 59, 34, 44, 44, 52, 35, 43, 30 $\bar{x} = 46,2; s = 14,5$
schwere Frösche	91, 111, 05, 82, 59, 52, 61, 70, 58, 90, 31, 64, 99, 71, 76, 64, 92, 74, 74, 76, 72, 64, 108, 89, 65, 89, 64, 69, 83, 61 $\bar{x} = 75,4; s = 17,8$	31, 56, 49, 79, 44, 45, 43, 56, 69, 40, 107, 38, 68, 79, 37, 65, 45, 74, 51, 113, 48, 125, 50, 58, 52, 117, 58, 45, 86, 60 $\bar{x} = 62,9; s = 24,9$

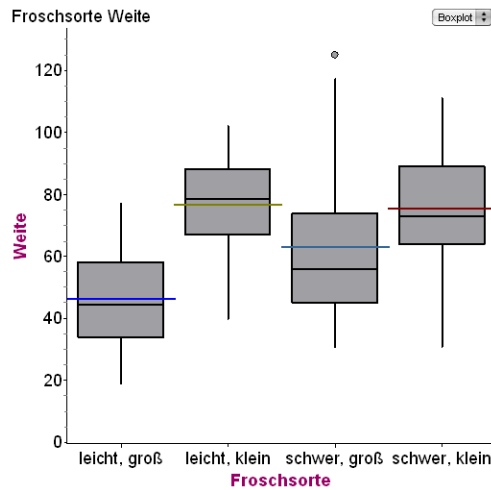


Classroom activities 11-12 grade

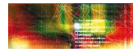
Descriptive analyses:

graphically focused by
a boxplot comparison:

Within the group of the
big paperfrogs weight
of paper has a stronger
influence than within
the group of the small
paperfrogs.



Markus Vogel, University of Education Heidelberg



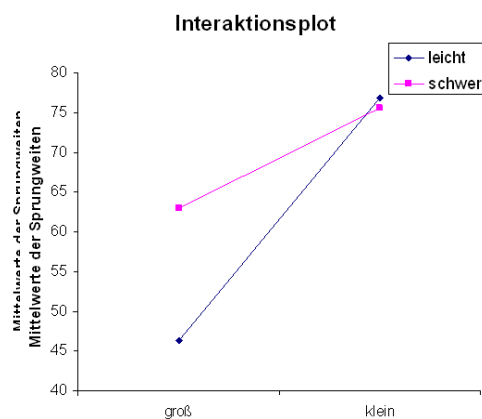
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Classroom activities 11-12 grade

Descriptive analyses:

graphically analyzed by
an interaction plot:

The interaction plot
shows a crossover
interdependence
between the variables
paper weight and paper
size



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Classroom activities 11-12 grade

Descriptive analyses:

numerical analyzes:

	small	big	total
lightweight	76,7	46,2	61,4
heavyweight	75,4	62,9	69,2
total	76,1	54,5	56,3

Influence power of factors:

Mean values

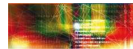
$$F_{paper\ weight} = \frac{1}{2} \cdot (69,2 - 61,4) = 3,9$$

$$F_{paper\ size} = \frac{1}{2} \cdot (76,1 - 54,5) = 10,8$$

Interdependence of factors measured by mean differences:

$$D_{big\ frogs} = 62,9 - 46,2 = 16,7$$

$$D_{small\ frogs} = 75,4 - 76,7 = -1,3$$



Classroom activities 11-12 grade

long term objective: inferential statistics

Simulation with regard to the distribution of the mean differences (with clustered data)



Chi-Square-Test (with clustered data):

$$\chi^2 = \sum_i \sum_j \frac{(e_{ij} - f_{ij})^2}{f_{ij}}$$



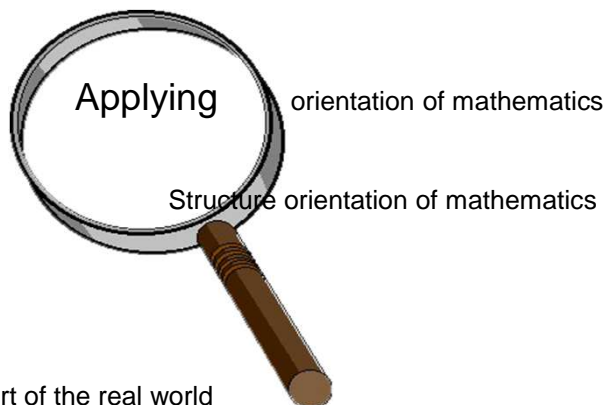
Überblick

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Curriculares

TIMSS, PISA & Co: *mathematical literacy*



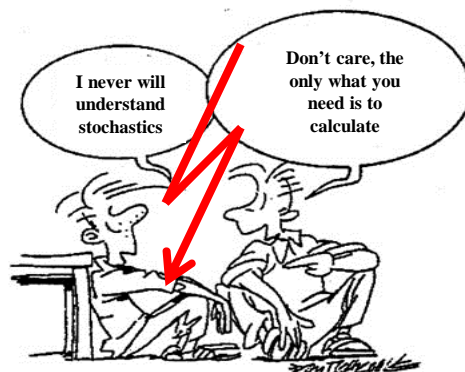
Hans Freudenthal:

- mathematics as part of the real world
- mathematics for understanding phenomena of the real world by reflecting on their underlying structure

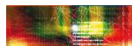


Curriculares

Also:



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Überblick

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Zusammenfassung

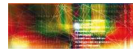
Aspects of statistical thinking (Pfannkuch/Wild, 1999)

1. Recognition of need of data



Data are founding an objective base

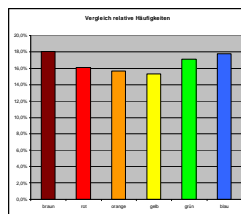
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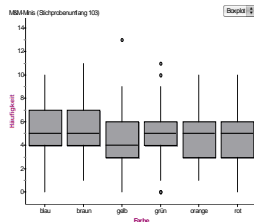
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Zusammenfassung

2. transnumeration



Changing representations to engender understanding



	braun	rot	orange	gelb	grün	blau
absolut	553	494	481	469	525	545
relativ	18,0%	16,1%	15,7%	15,3%	17,1%	17,8%

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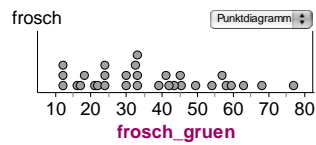
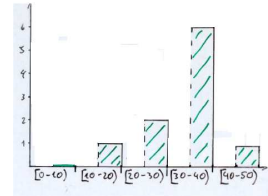
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Zusammenfassung

3. Consideration of variation

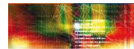
grüner Frosch (160g/cm³)

0	
1	8
2	3 8
3	0 1 1 2 2 7
4	1



measuring and modelling for the purposes of prediction, explanation, or control

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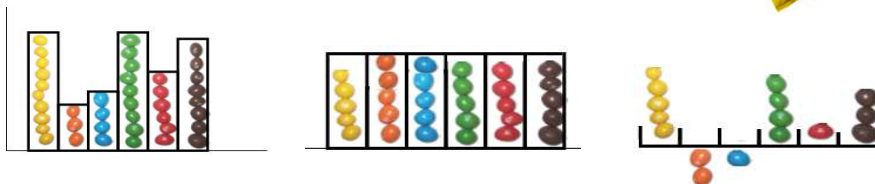
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Zusammenfassung

4. Reasoning with statistical models

data = pattern + deviation

is a construct to cope with omnipresent variability



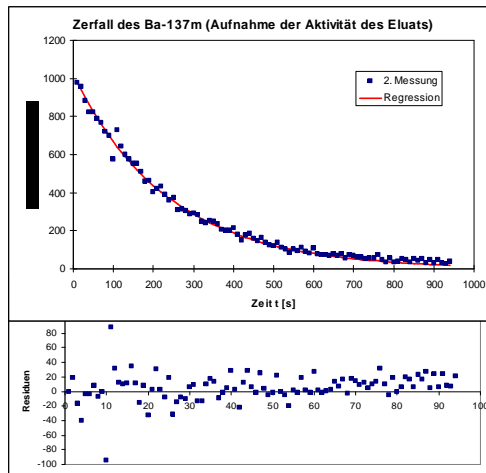
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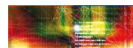
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Zusammenfassung

5. Integrating the statistical and contextual



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Leitidee Daten und Zufall - Mozilla Firefox

Thanks for your attention!

Leitidee Daten und Zufall

Mit den Bildungsstandards, welche die Kultusministerkonferenz im Jahr 2003 beschlossen hat, wurde die Leitidee Daten und Zufall verbindlicher Inhalt des Mathematikunterrichts für alle Bundesländer. Im Unterschied zur internationalen stochastikdidaktischen Diskussion nahm der Datenaspekt im deutschen Mathematikunterricht bis dahin gegenüber der Wahrscheinlichkeitsrechnung nur eine untergeordnete Rolle ein. Es stellt sich die Frage, wie die didaktische Schwerpunktverschiebung hin zu den Daten im Mathematikunterricht umgesetzt werden kann und wie Statistik und Wahrscheinlichkeitsrechnung zu der einen Leitidee Daten und Zufall verknüpft werden können.

Mit unseren Büchern und Lehrerfortbildungen haben wir uns zum Ziel gesetzt, mathematische, didaktische und unterrichtspraktische Antworten zu geben und so die Lehrenden im Bereich der Leitidee Daten und Zufall an Schule und Hochschule zu unterstützen. Uns ist bewusst, dass wir damit nur Ideen- und Hintergrundgeber für die Unterrichtsgestaltung vor Ort sein können, und wir hoffen, für Sie und den Mathematikunterricht gute und brauchbare Antworten gefunden zu haben.

Wir freuen uns über Ihre Rückmeldungen!
Andreas Eichler & Markus Vogel

eichlervogel@leitideedatenundzufall.de

www.leitideedatenundzufall.de

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