

Using ChatGPT in Mathematics Learning

SYMPOSIUM “AI AND CHATGPT IN EDUCATIONAL CONTEXTS”, 09.11.2023

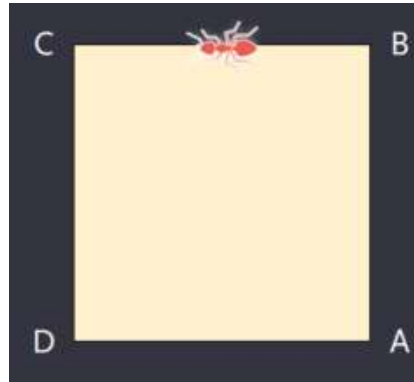
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Overview



How ChatGPT
works



ChatGPT and
mathematics

Let's go step by step.

Meaningful use
of ChatGPT

How ChatGPT works

- AI can learn probable combinations of data by seeing many examples
 - For example, it can be „trained“ on images that have associated descriptions.
- Given only partial data, the probability distribution of the missing data can be calculated
 - For example, If only the image is given, it can give probabilities for all possible descriptions

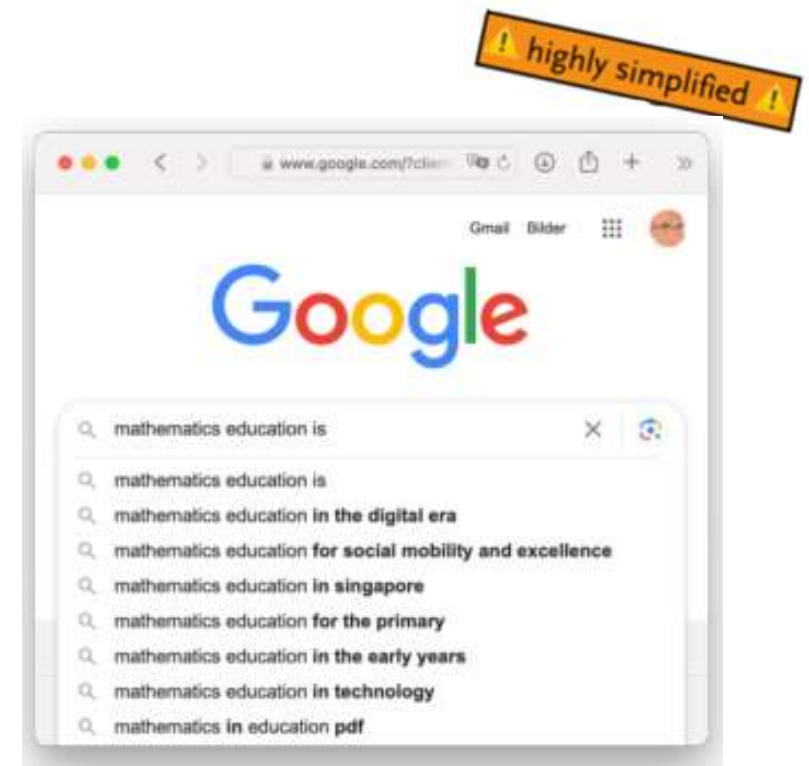


horse	42%
dog	52%
cat	6%

! highly simplified !

How ChatGPT works

- AI can learn probable combinations of data by training with many examples
 - For example, it can be trained on sentences taken from a huge corpus
- Given only partial data, the probability distribution of the missing data can be calculated
 - For example, in only the beginning of the sentence is given, it can give probabilities for all possible continuations



(Kortenkamp, 2023)

How ChatGPT works

- ChatGPT is a language model that generates texts based on stochastic information
- Outputs are not based on truth but on probability
- Responses can be regenerated and provide different outputs each time


ChatGPT and mathematics




- Novelty compared to calculators, spreadsheets, CA systems, ...:
 - Word problems can now be solved directly
 - No prior modeling is necessary
 - Different solutions can be produced
- Simple and possibly more complicated calculations are possible
 - By integration of Wolfram Alpha, also usable for higher mathematics
- Error-proneness
 - For example with large numbers
 - Texts are not / incorrectly understood
 - Algorithms are adopted hastily

(Spannagel, 2023; Kortenkamp, 2023; Bryden, 2023)

ChatGPT and mathematics

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A box contains 6 bananas and 2 kilos of apples. If each apple weighs 200 grams, how many fruits are inside the box? 

There are 2 kilos of apples, which is equal to $2 * 1000 = 2000$ grams.
There are 6 bananas, so the total is 6 bananas.
In total there are $2000 + 600 = 2600$ grams or 2.6 kilos of fruit in the box.   

(Bryden, 2023)

A box contains 6 bananas and 2 kilos of apples. If each apple weighs 200 grams, how many fruits are inside the box?

To find the total number of fruits in the box, you need to calculate the number of apples and bananas separately and then add them together.



First, let's calculate the number of apples:

2 kilos of apples = 2,000 grams of apples (since 1 kilo = 1,000 grams)

If each apple weighs 200 grams, then the number of apples is:

Number of apples = Total weight of apples / Weight of each apple

Number of apples = 2,000 grams / 200 grams/apple = 10 apples

Now, let's calculate the number of bananas:

There are 6 bananas in the box.

Now, add the number of apples and bananas together to find the total number of fruits:

Total number of fruits = Number of apples + Number of bananas

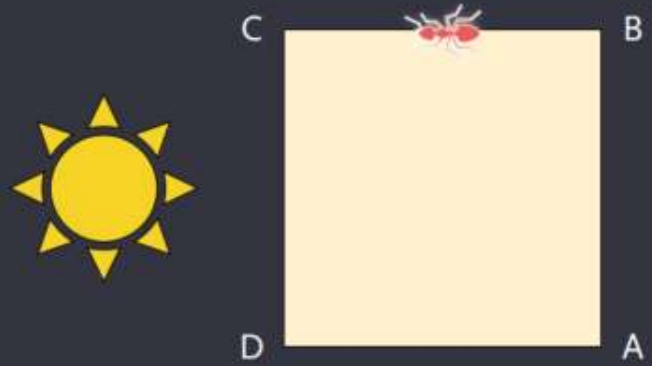
Total number of fruits = 10 apples + 6 bananas = 16 fruits

So, there are 16 fruits inside the box.

ChatGPT and mathematics

- Error-proneness
 - For example with large numbers
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 - Algorithms are adopted hastily

Problem Solving with ChatGPT



A square ABCD has a side length of 200 meters. An ant travels on the sides of the square starting at A. At daytime it travels 200 meters, at night it will be set back by half of its progress on the day. On which day will it return to A?

(Kortenkamp, 2023)

A square ABCD has a side length of 200 meters. An ant travels on the sides of the square starting at A. At daytime it travels 200 meters, at night it will be set back by half of its progress on the day. On which day will it return to A?

Let's break down the ant's journey to determine on which day it will return to point A.



Day 1:

During the daytime, the ant travels 200 meters from point A to point B.

Night 1:

At night, it is set back by half of its daytime progress, so it moves $200 \text{ meters} / 2 = 100$ meters backward from point B to point C.

Day 2:

During the daytime, the ant again travels 200 meters, moving from point C to point D.

Night 2:

At night, it is set back by half of its daytime progress, so it moves $200 \text{ meters} / 2 = 100$ meters backward from point D to point A.

Now, the ant has returned to point A at the end of Night 2. Therefore, the ant will return to point A on the second night, which means it will be back at A on the end of Day 2.

Meaningful use of ChatGPT

- ChatGPT can be used ...
 - to gather initial information on a particular topic, which should be verified later
 - for literature research
 - to learn how to ask the right questions (e.g., when having trouble understanding homework/learning material)
 - to support the development of professional skills of (prospective) mathematics teachers by anticipating learners' responses
 - to enhance Prompt Engineering

(Gíslason, 2023; Kortenkamp, 2023; Schorcht et al., 2023)

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(Gíslason, 2023; Kortenkamp, 2023; Schorcht et al., 2023)

Development of professional skills

- Lesson planning
 - Consider the relevant features of a lesson
 - Use Prompt Engineering to optimize lesson planning
- Learning process
 - ChatGPT as a learner who needs assistance in solving tasks
 - Use Prompt Engineering to facilitate learning

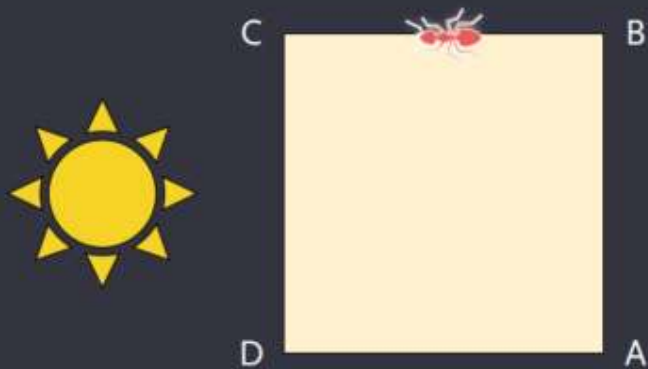
(Buchholtz & Huget, 2023; Gíslason, 2023; Kortenkamp, 2023; Schorcht, 2023)

Prompt Engineering

- Different ways of prompting:
 - Zero-shot-Learning
 - Few-Shot-Learning
 - Chain-of-Thought-Prompting
 - Ask-me-Anything-Prompting
- ChatGPT is slowed down with more detailed prompts, minimizing errors
- Optimizing the output through Prompt Engineering

(Schorcht et al., 2023; Ramlochan, 2023; Buchholtz & Huget, 2023; Wei et al., 2023; Arora et al., 2022)

Problem Solving with ChatGPT



A square ABCD has a side length of 200 meters. An ant travels on the sides of the square starting at A. At daytime it travels 200 meters, at night it will be set back by half of its progress on the day. On which day will it return to A?

Zero-shot-Learning

- prompt without giving further input

I'll give you an example for the first 2 days: On day 1 the ant travels 200 meters from point A, so at the end of the day it is on point B. During night time it travels 100 meters backwards, so it ends up between point A and point B. During daytime on day 2 it travels 200 meters and ends up between point B and point C. During night time it goes 100 meters backwards and ends up at point B. Can you continue?

I see the pattern now, and I appreciate your patience. Let's continue the ant's journey based on your description:



Day 3:

- The ant starts at point B.
- During the daytime, it travels 200 meters from point B to point C.

Night 3:

- At night, the ant is set back by half of its daytime progress, which is $200 \text{ meters} / 2 = 100 \text{ meters}$.
- So, it moves 100 meters backward from point C to point B.

Day 4:

- The ant starts at point B.
- During the daytime, it travels 200 meters from point B to point C.

Few-Shot-Learning

- give a concrete example
- „I'll give you an example...”

Night 4:

- At night, the ant is set back by half of its daytime progress, which is 200 meters / 2 = 100 meters.
- So, it moves 100 meters backward from point C to point B.

This pattern continues:

Day 5:

- The ant starts at point B.
- During the daytime, it travels 200 meters from point B to point C.

Night 5:

- At night, the ant is set back by half of its daytime progress, which is 200 meters / 2 = 100 meters.
- So, it moves 100 meters backward from point C to point B.

The ant keeps going back and forth between points B and C during the daytime and nighttime, and it will not return to point A. Therefore, the ant will never return to point A in this scenario.

Let's go step by step.

Of course, let's break down the ant's journey step by step:



Day 1:

- The ant starts at point A.
- During the daytime, it travels 200 meters from point A to point B.

Night 1:

- At night, it is set back by half of its daytime progress.
- So, it moves $200 \text{ meters} / 2 = 100 \text{ meters}$ backward from point B to point C.


Day 2:

- During the daytime, the ant travels 200 meters from point C to point D.

Night 2:

- At night, it is set back by half of its daytime progress.
- So, it moves $200 \text{ meters} / 2 = 100 \text{ meters}$ backward from point D to point A.

Now, the ant has returned to point A at the end of Night 2. Therefore, the ant will return to point A on the second night, which means it will be back at A at the end of Day 2.

 Regenerate

Chain-of-thought-Prompting

- previous output is used and refined

A square ABCD has a side length of 200 meters. An ant travels on the sides of the square starting at A. At daytime it travels 200 meters, at night it will be set back by half of its progress on the day. On which day will it return to A? Ask me anything you need to solve this task.



Thank you for your patience. Let's solve this step by step.



Day 1:

- The ant starts at point A.
- During the daytime, it travels 200 meters from point A to point B.

Night 1:

- At night, the ant is set back by half of its daytime progress.
- So, it moves $200 \text{ meters} / 2 = 100 \text{ meters}$ backward from point B to point C.

Day 2:

- The ant starts at point C.
- During the daytime, it travels 200 meters from point C to point D.

Night 2:

- At night, the ant is set back by half of its daytime progress.
- So, it moves $200 \text{ meters} / 2 = 100 \text{ meters}$ backward from point D to point A.

Now, the ant has returned to point A at the end of Night 2. Therefore, the ant will return to point A on the second night, which means it will be back at A at the end of Day 2.

Ask-me-anything-Prompting

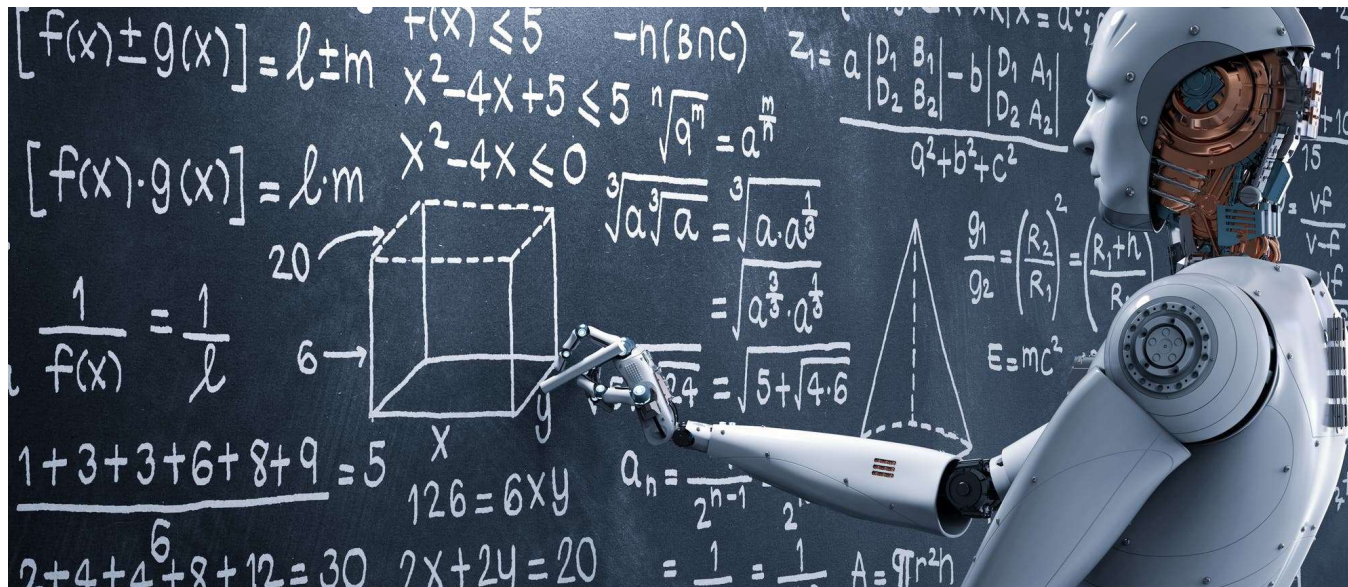
- Generation of questions to solve the task

Conclusion

- Rapid development
- What ChatGPT cannot do
 - Reproduce emotions (frustration, elation, confusion...)
 - No evaluation of correct/incorrect answers
 - Does not insist on the answer
 - Mirror the actual reaction of children
- Potentials for mathematics teaching and learning
 - Making errors of AI usable for the professional development of (prospective) teachers
 - Support lesson planning
 - Refine Prompt Engineering

Thank you for your attention!

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Literature

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